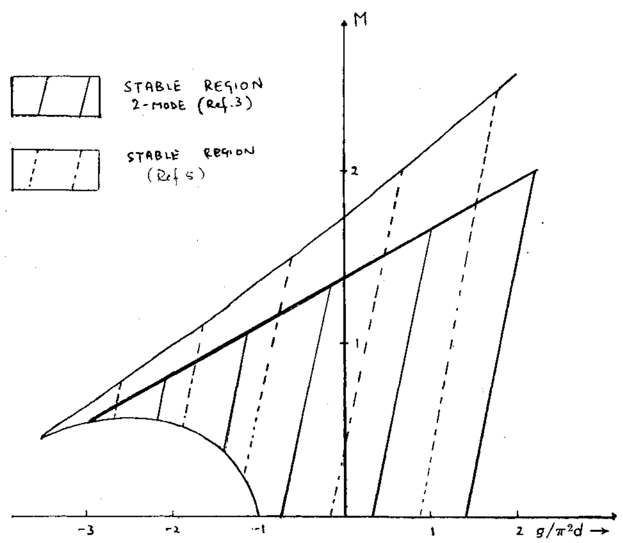

Fig. 1 Stability boundaries.

By using the functional

$$f = -\int_0^1 [dz_{,xx}{}^2 + gz_{,x}{}^2 + \mu\dot{z}^2]dx$$

it is possible to show that the panel is unstable for $M = 0$ if $g < -\pi^2 d$. However, the two-mode analysis of Johns and Parks,[3] a searching analysis of Movchan[4] and further work by Pritchard[5] all indicate that for some values of the tension less than the buckling load $(-\pi^2 d)$ there is a range of values of the Mach number for which the panel is stable. The accompanying results are computed for $\mu = 200\ d = \frac{1}{200}$ (see Fig. 1).

References

[1] Webb, G. R. et al., "Further Study on 'A Stability Criterion for Panel Flutter via the Second Method of Liapunov,' " *AIAA Journal*, Vol. 5, No. 11, Nov. 1967, pp. 2084–2085.

[2] Parks, P. C., "A Stability Criterion for Panel Flutter via the Second Method of Liapunov," *AIAA Journal*, Vol. 4, No. 1, Jan. 1966, pp. 175–177.

[3] Johns, D. J. and Parks, P. C., "The Effect of Structural Damping on Panel Flutter," *Aircraft Engineering*, Vol. 32, 1960, pp. 304–308.

[4] Movchan, A. A. "On the Stability of a Panel Moving in a Gas," *Prikl. Mat. i Mekh.*, Vol. 21, No. 2, 1957.

[5] Pritchard, A. J., *AIAA Journal* (to be published).

Reply by Authors to R. H. Plaut, and to P. C. Parks and A. J. Pritchard

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Charles H. Goodman,† Malcolm A. Goodman,†
and Karl M. Land†
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THE Comments by R. H. Plaut and by T. C. Parks and A. J. Pritchard are certainly correct. We appreciate their pointing out this bit of carelessness.

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† Graduate Student.

Comment on "A Comparative Study of Numerical Methods of Elastic-Plastic Analysis"

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PROFESSOR Marcal[1] claims to have shown that the initial strain method, which uses the constant strain approach, breaks down for elastic perfectly plastic materials. In order to demonstrate the claim to be false, the constant strain approach is used to obtain the solution to the symmetrical three-bar truss shown in Fig. 1.

An elastic perfectly plastic material of modulus E and yield strength σ_o is assumed. The area of each of the pin-connected truss members is denoted by A, the downward load at joint 1 by P, and the corresponding deflection by x. In the elastic range of material behavior the load displacement relationship in terms of the dimensionless quantities $\bar{x} = Ex/H\sigma_o$ and $\bar{P} = P/A\sigma_o$ is

$$(1 + a)\bar{x} = \bar{P} \tag{1}$$

where

$$a = 2\sin^3\phi \tag{2}$$

and where H is the depth of the truss and ϕ the slope angle of the inclined truss members. Equation (1) holds for values of $\bar{x}$ in the range $0 \leq \bar{x} \leq 1$, that is, for $\bar{P} \leq 1 + a$. For larger values of $\bar{P}$ the center bar of the truss becomes fully plastic while the inclined bars remain elastic until the collapse load P_c is reached, where

$$\bar{P}_c = 1 + 2\sin\phi \tag{3}$$

For values of $\bar{P}$ in the range $1 + a \leq \bar{P} \leq \bar{P}_c$ the load-displacement relationship can be written in the form

$$(1 + a)\bar{x} = \bar{P} + (\bar{x} - 1) \tag{4}$$

Equation (4) is the initial strain method formulation. The bracketed term on the right-hand side of Eq. (4) can be thought of as a fictitious load that, when added to the actual load, accounts for the effects of plastic flow in the vertical bar. The so-called tangent modulus formulation is obtained by eliminating the displacement from the right-hand side of Eq. (4) to give

$$a\bar{x} = \bar{P} - 1 \tag{5}$$

Solution of Eq. (4) by the following iterative scheme is called the constant strain method:

$$\bar{x}^{(n)} = \frac{1}{1 + a}[\bar{P} - 1 + \bar{x}^{(n-1)}] \qquad n = 1, 2, \ldots \tag{6}$$

in which $\bar{x}^{(n)}$ is the nth iterate and $\bar{x}^{(o)} = 1$. An explicit expression for the value of the nth iterate can be obtained from Eq. (6) as

$$\bar{x}^{(n)} = [(\bar{P} - 1)/a][1 - 1/(1 + a)^n] + 1/(1 + a)^n \tag{7}$$

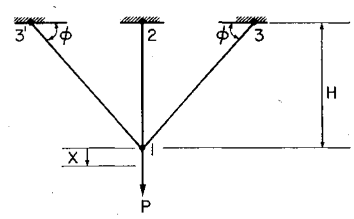

Fig. 1 Symmetrical three-bar truss.

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Since the tangent modulus stiffness a is positive, it is evident that, as n increases, the value of $\bar{x}^{(n)}$ approaches the exact value given by Eq. (5).

Reference

[1] Marcal, P. V., "A Comparative Study of Numerical Methods of Elastic-Plastic Analysis," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 157–158.

Reply by Author to M. A. Salmon

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THE lack of convergence of the constant strain approach for an elastic perfectly plastic material was argued for a two-dimensional constant stress element.[1] It is not possible to use Salmon's example of a one-dimensional constant stress element to test the above claim, since the material exhibits different behavior in the two different cases. Because of the normal flow rule of plasticity, the yielded material in two dimensions still possesses a certain amount of resistance to straining. This is reflected in the stress-strain relation $[P^-]$. This is not true in the one-dimensional case where $[P^-]$ is equal to zero.

Salmon's criticism does, in fact, raise an important point. Since most of the analysis was developed in general matrix form, it would be expected that, for an elastic perfectly plastic material, the lack of convergence should apply equally to all types of constant stress elements. However, this line of reasoning neglects the fact that $[P^-]$ does not exist for a one-dimensional truss element. Because $[P^-]$ does not exist, Eq. (13) of Ref. 1 is invalid, and the convergence study that is based on this equation can no longer be expected to hold.

Reference

[1] Marcal, P. V., "A Comparative Study of Numerical Methods of Elastic-Plastic Analysis," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 157–158.

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Comment on "A Formula for Updating the Determinant of the Covariance Matrix"

JAMES E. POTTER* AND DONALD C. FRASER†
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A FORMULA was presented in Ref. 1 for updating the determinant of the covariance matrix of state estimation errors when measurement statistics are incorporated. This result may be obtained in a much simpler way if a more general identity is first proved. This general identity is

$$|A||D + CA^{-1}B| = |D||A + BD^{-1}C| \tag{1}$$

where the individual matrices have the following form:

$A = n \times n$ (nonsingular) $\quad C = m \times n$

$B = n \times m \quad D = m \times m$ (nonsingular)

This identity is obtained by manipulating partitioned matrices as follows:

$$\begin{bmatrix} A & -B \\ C & D \end{bmatrix} \begin{bmatrix} I & A^{-1}B \\ O & I \end{bmatrix} = \begin{bmatrix} A & O \\ C & (D + CA^{-1}B) \end{bmatrix} \tag{2}$$

Since the second matrix on the left side of Eq. (2) has unity determinant, there results

$$\begin{vmatrix} A & -B \\ C & D \end{vmatrix} = |A|\ |D + CA^{-1}B| \tag{3}$$

Similarly,

$$\begin{bmatrix} A & -B \\ C & D \end{bmatrix} \begin{bmatrix} I & O \\ -D^{-1}C & I \end{bmatrix} = \begin{bmatrix} A + BD^{-1}C & B \\ O & D \end{bmatrix} \tag{4}$$

$$\begin{vmatrix} A & -B \\ C & D \end{vmatrix} = |D|\ |A + BD^{-1}C| \tag{5}$$

Equating Eqs. (3) and (5) gives the identity of Eq. (1). Using the following change of vocabulary in Eq. (1):

$$A = P'^{-1} \qquad B = H \qquad C = H^T \qquad D = R \tag{6}$$

together with the optimum linear filter update equation

$$P^{-1} = P'^{-1} + HR^{-1}H^T \tag{7}$$

leads directly to the main result of Ref. 2,

$$|P|/|P'| = |R|/|R + H^TP'H| \tag{8}$$

By making the substitutions

$$D = I \qquad C = DC \tag{9}$$

Eq. (1) can be extended to the case where D is singular. The result is

$$|A|\ |I + DCA^{-1}B| = |A + BDC| \tag{10}$$

Reference

[1] Potter, J. E. and Fraser, D. C., "A Formula for Updating the Determinant of the Covariance Matrix," *AIAA Journal*, Vol. 5, No. 7, July 1967, pp. 1352–1354.

Received March 28, 1968.
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Comment on "Feasibility of a High-Performance Aerodynamic Impulse Facility"

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THE entropy correlation of the nonequilibrium chemical species concentrations of expanding air offered in Refs. 1 and 2 was based upon l values [i.e., $A/A^* = 1 + (x/l)^2$] of only 1 and 4.74 cm. Therefore, this correlation as reproduced in Ref. 3 is in error. The value of $l = 10$ cm shown in Ref. 3 appears to be a typographical error introduced when

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